

PRELIMINARY STUDY OF FRICTIONAL POWER LOSSES IN SPUR GEARED TRANSMISSIONS

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ABSTRACT

Friction phenomenon in gear systems has been the subject of several studies in the last decades, and it is still a very active area of research. The complexity of the friction phenomenon, occurring when two teeth are in contact during the meshing phase, results in the fact that, many gear key parameters vary simultaneously essentially the velocities, the friction coefficient and the transmitted load. Due to the rising of fuel prices and the increase in energy demand on one hand, and the governmental regulations and environmental pressures in terms of the allowable gases and particulates released to the environment which are becoming more and more strict on the other hand, there is a continuous demand for higher efficiency gears. The objective of this work is to contribute to a better understanding of the power losses due to the friction phenomenon in external spur gear sets, which can be grouped into two categories; sliding and rolling power losses. In this preliminary study, we present an analytical method to obtain the sliding power losses. In order to understand the engagement between the pinion and the wheel along the line of action, a geometric study was carried out and was used to instantly calculate different parameters

that lead to the estimation of the sliding power losses. The rolling phenomenon will be discussed in our future work, since the total power losses related to friction is the sum of the sliding and rolling power losses.

KEYWORDS: *Spur Gear, Line of action, Sliding Velocity, Coefficient of Friction Sliding Power Losses*

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INTRODUCTION

Gears are mainly used to transmit power, load and a rotational motion from one shaft to another. During this process, some of the power is inevitably lost due to the friction in the power transmitters. Several applications including machine tool, automotive industry and in industrial gear boxes use gears to accommodate the power according to the needs. Their popularity is linked to better performance they provide, but the increase in power to be transmitted at high rotational speeds results in significant power losses in geared transmissions, then since, power losses have direct impact on the life time of the power transmitters, they must therefore be taken into account during the different phases, especially when designing and choosing the materials of the gears. A well designed mechanical system would enable the user to avoid breakage of the mechanism due to thermal expansion, and would allow a better design of the cooling systems. Besides, the energy costs continue to rise, therefore, it is very important to estimate the total power losses, since they have a direct impact on the energy losses.

To understand the friction phenomenon, some studies were carried out, but many of them studied the friction coefficient, since the latter is directly linked to the power losses resulting to sliding between spur gear teeth. We can cite for example Drozdov and Gavrikov [1], Benedict and Kelley [2], O' Donoghue and Cameron

[3], Misharin [4]; these authors proposed empirical equations to determine the friction coefficient. Most of these empirical laws were obtained by simulating the gear meshing, and in some cases that resulted in shortcomings, since their experimental set ups didn't take into account all of the parameters in contact, such as the surface roughness etc. For example, the Benedict and Kelley's friction law [2] has \log_{10} in its analytical expressions, which lead to an infinite friction at the pitch point where, there is no relative sliding and it underestimates the friction coefficient for high slide to roll conditions. Considering tooth friction, Diab et al [5] proposed a traction law based on measurements from a two-disc machine, which accounts for lubricant properties and surface, finish and integrated in a 3D dynamic model of gears. By simulating their model using a constant Kelley's frictional coefficient, they came to the conclusion that, it yields best results for both low and higher speeds. After having performed a multiple linear regression analysis to an elastohydrodynamic lubrication model, Xu and al [6] proposed a friction coefficient formula that takes into account many parameters in the contact between spur gear teeth. In order to estimate the power losses and efficiency in spur geared transmissions, Anderson and Loewenthal [7] studied the case of a steel spur gear set of an arbitrary geometry, supported by ball bearings at part and full load. They used the Benedict and Kelley's [2] friction coefficient formula to calculate the sliding power losses.

Friction is defined as the resistance to motion between two surfaces in relative sliding and rolling under dry or lubricated contact conditions. Friction losses largely depend on sliding and rolling velocities and load. The aim of this work is therefore to summarize, discuss and analyze the advances on modelling approaches developed for the estimation of friction power losses in spur geared transmissions. We start by first describing the friction phenomenon, then the friction key parameters will be discussed, so as to introduce the details on how to analytically calculate sliding power losses along the line of action. Finally, some insights for future works will be suggested

NOMMENCLATURE

b	Face width of tooth (mm)
C	torque (Nm)
Σ	contact ratio
db1	diameter of the pinion base circle (mm)
db2	diameter of the wheel base circle (mm)
dp	pitch diameter (mm)
dt1	diameter of the pinion dedendum circle (mm)
dt2	diameter of the wheel dedendum circle (mm)
i	transmission ratio
L	length of the line of action (mm)
N	rotational speed (rpm)
m	module (mm)
ν_0	dynamic viscosity (cPo)

μ	Coefficient of friction
p_h	Maximum pressure of Hertz (Pa)
P_s	sliding power loss (Kw)
r_{p1}	pinion pitch radius (mm)
r_{p2}	wheel pitch radius (mm)
R	equivalent radius of curvature (mm)
S	surface roughness parameter
θ	pressure angle (deg)
V_e	entraining velocity (m.s-1)
V_r	rolling velocity (m.s-1)
V_s	sliding velocity (m.s-1)
W	gear contact normal load (N)
z_1	number of the teeth of the pinion
z_2	number of the teeth of the gear

Description of the Friction Phenomenon in Spur Geared Transmissions

In a geared transmission, the total power loss can be divided into two categories: load and speed-dependant losses Figure (1). Speed-dependant losses consist of windage losses due to the interaction with the air surrounding the gear, churning losses due to the entrapment of the lubricant in gear mesh and seal losses. Load-dependant losses are made up bearing loss and friction losses. In function of the operating conditions, each power loss category controls the overall efficiency of the system. As cited in [5] and in [8], the friction phenomenon related power losses are preponderant at low speeds, where as they are low at high-speed in comparison to the power losses due to the ventilation phenomenon and to the trapping phenomenon, which was discussed by the authors in [9] and in [10].

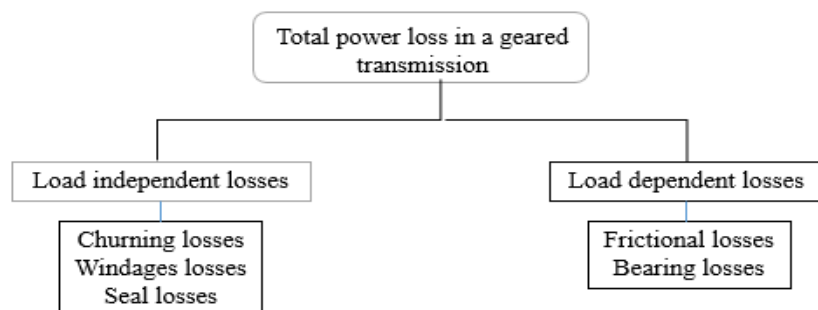


Figure 1: Sources of Power Losses in a Geared Transmission

The applied lubricant to a contact significantly alters the contact conditions, and hence friction. Applied load, speed, parameters related to contact geometry, surface roughness and lubricant all help define the lubrication conditions [6]. Gears are usually operated under mixed Elastohydrodynamic Lubrication (EHL) conditions, where the lubricant film thickness is comparable to the surface asperity heights, such that, actual metal-to-metal contacts are possible. Therefore,

the friction between the gear teeth can be considered as a mixture of dry and fluid friction. For a better understanding of the friction phenomenon resulting from metal-metal in non-lubricated contacts also known as dry friction, many experiments were conducted. In the case of lubricated contacts, there are few models to identify qualitatively and quantitatively the friction phenomenon, and its direct influence on power loss in geared power transmissions. According to Diab [5], Velex P. and Cahouet, V. [8], the friction between teeth is one of the major sources of power dissipation and may also be a source of vibrations and noises. The total power losses resulting from this phenomenon is closely linked to the coefficient of friction.

Distances along the Line of Action

The line of action is a straight line for involute gear geometry in space, mapped out of point of contact of two gears in mesh. The points x_a to x_d shown in the figure 4 represent the following sequence of events;

x_a : start of mesh cycle also known as start of active profile (SAP), two teeth share the load,

x_b : Start of single-tooth profile (LPSTC),

x_p : Pitch point (P),

x_c : End of single tooth-pair contact (HPSTC),

x_d : End of active profile (EAP).

Those points allow us to instantly know the position of the meshing position and they are defined, in function of gear parameters as follows [7]:

Start of action line (SAP):

$$x_a = \frac{M (Z_1 + Z_2) \sin \theta}{2} - \sqrt{\left(\frac{dt_2}{2}\right)^2 - \left(\frac{db_2}{2}\right)^2} \quad (1)$$

Start of single tooth-profile meshing of a pair of teeth (LPSTC):

$$x_b = x_d - \pi m \cos \theta \quad (2)$$

Pitch point:

$$x_p = \frac{dp \sin \theta}{2} \quad (3)$$

End of single tooth-pair contact (HPSTC):

$$x_c = x_a + \pi m \cos \theta \quad (4)$$

End of active profile (EAP):

$$x_d = \sqrt{\left(\frac{dt_1}{2}\right)^2 - \left(\frac{db_1}{2}\right)^2} \quad (5)$$

Length of the Line of Action and the Contact Ratio

In function of geometric parameters, the length of the line of action along which the load is transmitted is given

by:

$$L = \sqrt{(rp_1 + m)^2 - (rp_1 \cos \theta)^2} + \sqrt{(rp_2 + m)^2 - (rp_2 \cos \theta)^2} - (rp_1 + rp_2) \sin \theta \quad (6)$$

The contact ratio is the ratio between the length of the contact path and the base pitch and is the number, the teeth is in contact along the path of contact.

$$\Sigma = \frac{L}{\pi m \cos \theta} \quad (7)$$

The geometric and operational data used in this study are cited in the table 1 below:

Table 1: Geometric and Operational Data Used in This Study [7]

Parameter	Value
Number of teeth for pinion	48
Number of teeth for gearModule (mm)	80
Face width of tooth (mm)	3.175
Pressure angle (deg)	40
Torque (Nm)	20
Rotational speed (rpm)	271
Effective radius of curvature (mm)	2000
Maximum pressure of Hertz (Gpa)	2
Dynamic viscosity (cPo)	13
Surface roughness parameter (μm)	0.07

In the table 2 below, we presented the position of characteristic points along the line of action using the geometric and operational data, as given in table 1. Considering that the load is transmitted from pinion to wheel, the origin of our axis is the intersection of the path of action, with the base circle of the pinion. The line of action is the distance between SAP and EAP as cited above, the contact ratio define the number of gear pair teeth in contact. In this study, the contact ratio is found to be 1.78, which means that there is a possibility to have two gear pair teeth in contact and that was taken into consideration in the normal transmitted load model.

Table 2: Characteristic Points along the Line of Action

Point	value
Start of active profile (SAP)	17.50556
Start of single tooth-pair contact (LPSTC)	24.87867
Pitch point (P)	26.06193
End of single tooth-pair contact (HPSTC)	26.87858
End of active profile (EAP)	34.25164
Length of line action	16.74608
Contact ratio	1.78663

Sliding Velocity

In spur geared transmissions, the sliding velocity was defined by [7] as follow:

$$V_s(x) = \frac{0.1047(1+i)N(x-x_p)}{i} \quad (8)$$

The figure 2 below presents the variation of the sliding velocity along the line of action in function of geometric and operational parameters:

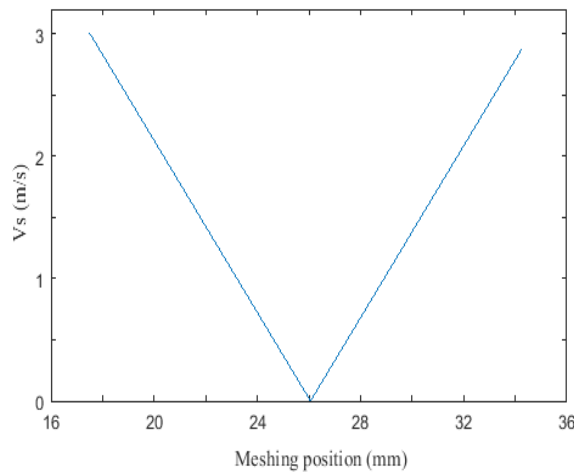


Figure 2: Sliding Velocity Along the Path of Contact

The maximum value of the sliding velocity is found at the starting and ending points of meshing, while its minimum value is obtained at the pitch point.

Sliding Force

The instantaneous frictional force due to sliding of two gears teeth against each other is [7]:

$$F_s(x) = \mu(x)W(x) \quad (9)$$

Coefficient of Friction

The coefficient of friction significantly affects the sliding power losses in a loaded gearset. There are several friction laws to estimate the latter, a comparative study of different friction laws was studied in [11]. In this work, we will use the empirical formula and the experimental one, in order to compare results in terms of sliding power losses.

Empirical Law of Friction

The Benedict and Kelley's friction law is used in this study.

$$\mu = 0.0127 \log_{10} \left(\frac{29,66W(x)}{v_0 b V_s (V_r)^2} \right) \quad (10)$$

This formula does not predict the accurate values of the friction coefficient at low sliding velocity at the pitch point, some authors like [7] took an upper limit of $\mu=1$ at that point, and justified their choice by the fact that the sliding power loss is relatively low there and of short duration.

Experimental Law of Friction

To establish a friction law that takes into account several meshing factors including lubrication, material type and surface roughness, the authors H.Xu, A. Kahraman, N. Anderson and D. Maddock [6] considered the contact between teeth as an elastohydrodynamic (EHD) contact because, the latter is characterized by low contact areas, significant surface deformation, high contact pressure and the presence of the oil film between the contacting asperities. They came to the following friction law:

$$\mu = e^{f(SR, p_h, v_0)} p_h^{c_2} |SR|^{c_3} V_e^{c_6} v_0^{c_7} R^{c_8} \quad (11)$$

$$f(SR, p_h, v_0) = c_1 + c_4 |SR| p_h \log_{10}(v_0) + b_5 e^{-|SR| p_h \log_{10}(v_0)} + c_9 e^S \quad (12)$$

Where: SR is the slide to roll ratio defined as the ratio between the sliding velocity and the rolling velocity, and the constants c_i are given as: $c_i = -8.916465$; 1.03303 ; 1.036077 ; -0.354068 ; 2.812084 ; -0.100601 ; 0.752755 ; -0.390958 ; 0.620305 for $i = 1$ to 9 .

Comparison Between Used Laws of Friction

In the case of the lubricated contacts, the coefficient of friction and the power loss due to friction vary with the load, the rotational speed, the sliding speed, the properties of the pinion, wheel materials and the properties of the used lubricants as well. Their variations during the meshing phase are shown on the figure 3 below:

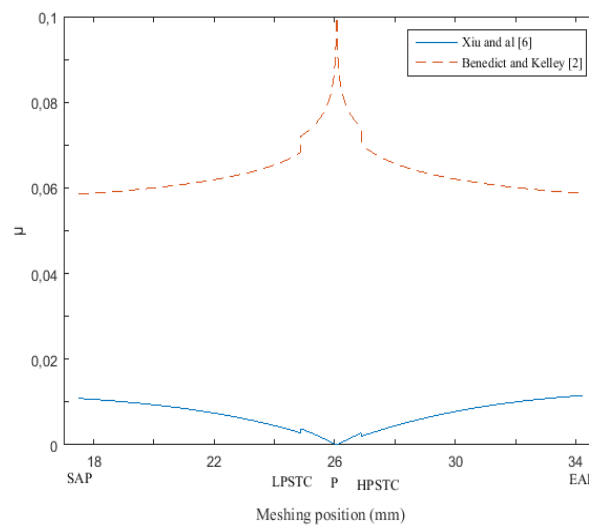


Figure 3: Variation of the Coefficient of Friction Along the Line of Action

It is noted that the empirical law [2] predicts a minimum value of the frictional coefficient at the meshing start and end points and a maximum value at the pitch point, while the experimental law [6] predicts a friction coefficient, whose maximum value is obtained at the meshing start and end points, and for which, the minimum value is found at the pitch point. This is consistent with the results published by the authors of reference [7] and [6].

Normal Transmitted Load

The normal load on the gear tooth surface may be calculated from the applied pinion torque as follow [7];

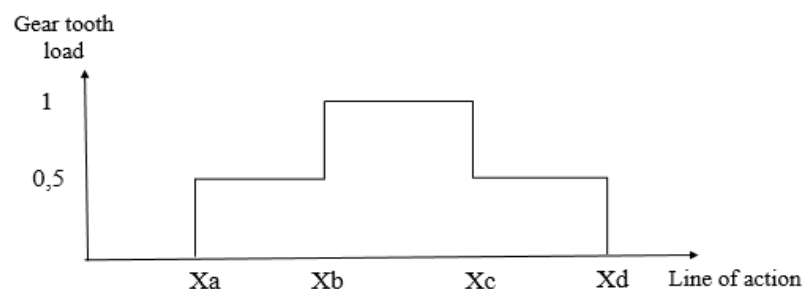


Figure 4: Load Evolution Along the Path of Contact

Between points Xb and Xc along the line of action, the load is carried by one pair of teeth;

$$W(x) = \frac{2C}{d_p \cos \theta} \quad (13)$$

Between points Xa, Xb and Xc, Xd along the line of action, the load is carried by two pair of teeth;

$$W(x) = \frac{C}{d_p \cos \theta} \quad (14)$$

On the the figure 5 below, we plotted the normal load to be transmitted from pinion to the wheel.

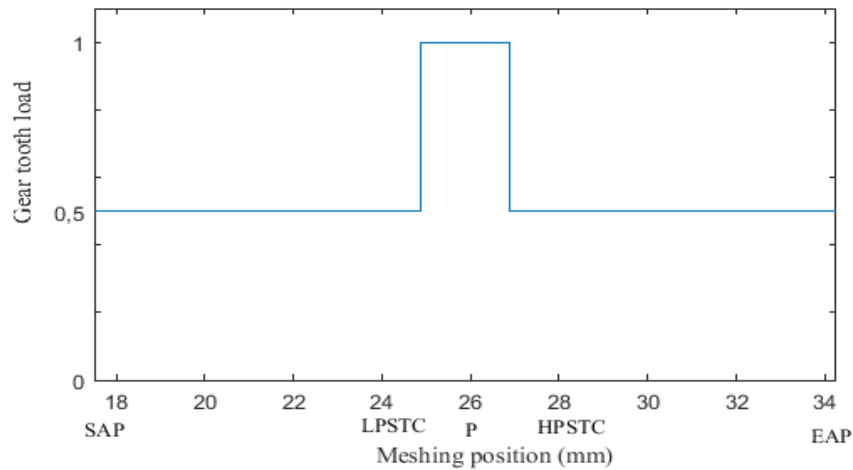


Figure 5 : Transmitted Load Along the Path of Contact

On the figure 6 below, we plotted the instantaneous sliding force using the two friction laws discussed above,

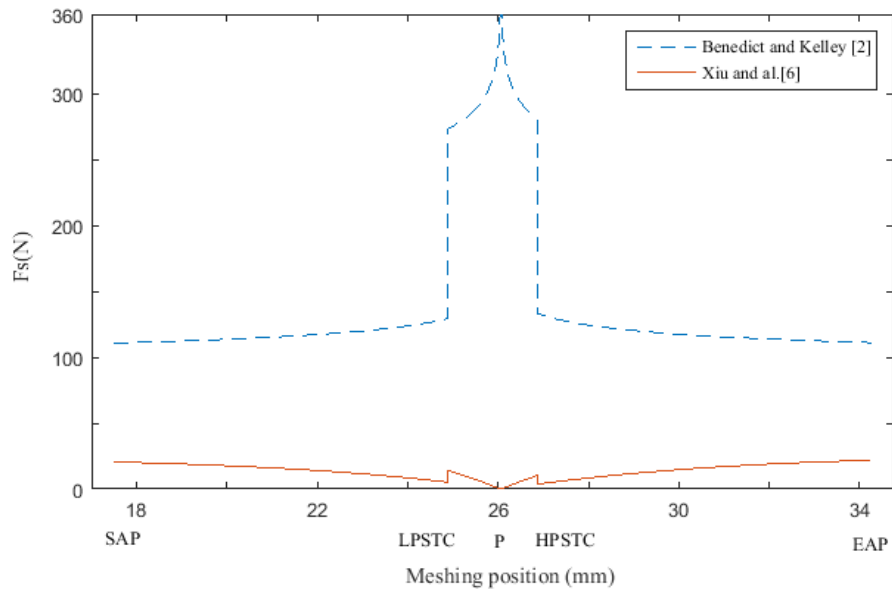


Figure 6: Sliding Force along the Path of Contact

By comparing the experimentally validated friction law [6] to the Benedict and Kelley's friction law [2], it is remarked that, the latter predicts its maximum value of the frictional force at the pitch point, where there is no sliding but as we shall see later, it correctly predicts its minimum value of the sliding power losses at that point, which is in agreement

with the results found using the Xiu et al's friction law [6].

Sliding Power Losses

The instantaneous power losses due to the friction between spur gear teeth are calculated by the product of the frictional force by the sliding speed.

$$P_s(x) = F_s(x)V_s(x) \quad (15)$$

On the figure 7 below, the sliding power losses is presented:

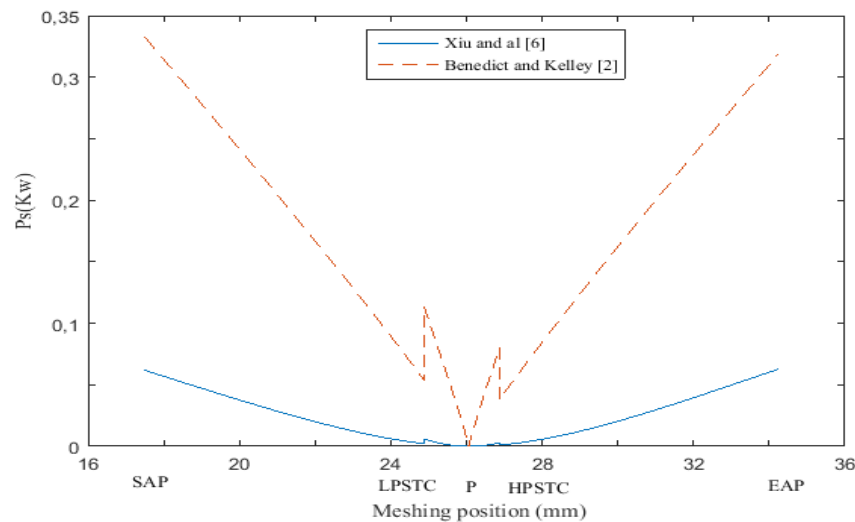


Figure 7: Variation of the Power Loss Along the Line of Action

The instantaneous variation of the sliding power loss presented on the figure 7 above, shows that both Xiu and al and Benedict and Kelley's friction predict zero sliding at the pitch point and elsewhere, the latter overestimates the power losses comparing to Xiu and al's law.

CONCLUSIONS

In spur geared transmissions, the power loss linked to the friction phenomenon can be divided into two contributions; one is the sliding between teeth and the other is the rolling, which is caused by hydrodynamic forces on the gear teeth. This paper aimed to present an analytical method to instantly calculate the sliding power losses in external spur gear sets. The rolling phenomenon will be studied in our future work, since the total friction power loss is the sum of sliding and rolling power losses. For a better understanding of what happens along the path of contact, which is the line along which the contact between gear teeth occurs. The characteristic points and distances along the path of contact were explained and their expressions were given and referenced. The sliding power losses depend on the friction coefficient, the transmitted load and the sliding velocity.

Many friction laws were presented, but some were not accurate in terms of today's operating conditions. Since establishing all parameters in the contact were not taken into consideration, a comparative study of friction laws was carried out [11] and allowed us to choose the Benedict and Kelley and Xiu and al friction law. It's because, despite the fact that these laws predict different results of the coefficient of friction at the pitch point, where, there is no sliding, they accurately predicted zero sliding power loss at that point, which was also remarked by [6] and [7]. The transmitted load

was expressed in terms of the torque and other geometric parameters, and by also taking into account, the contact ratio is defined as the average number of teeth in engagement.

The sliding velocity expression given in [11] allowed us to instantly know the variations of the latter and it was remarked that, its maximum values are obtained at the start (SAP) and at the end (EAP) meshing, whereas its minimum value is found at the pitch point.

It is observed that both Xiu and al and Benedict and Kelley's friction predict zero sliding at the pitch point, and elsewhere the latter over estimates the power losses comparing to Xiu and ali's law. In this preliminary study of friction power losses, we presented an analytical method to obtain the sliding power losses by outlining and referencing all parameters, and for better understanding, our future study of the rolling related power losses will be followed by a numerical integration, in order to quantify the total friction power losses, the impact of operational and geometric parameter on the latter and finally the gear efficiency related to the friction phenomenon.

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